APPENDIX A

A-1. NORMAL APPROXIMATION TO BINOMIAL

When p, the probability of failure on a given trial, is moderate ($0.2 \leq p \leq 0.8$) and n the number of trials is large ($n \geq 30$), the normal distribution provides reasonable approximations to binomial probabilities. This approximation is detailed below. Note that Z is the notation for the standard normal variable. (See Appendix B, Table 2.)

The probability of k or fewer failures out of n trials is approximately equal to

$$1 - P(Z \geq (k+0.5-np)/\sqrt{np(1-p)}) .$$

The probability of at least k failures out of n trials is approximately equal to

$$P(Z \geq (k-0.5-np)/\sqrt{np(1-p)}) .$$

The probability of between k_1 and k_2 failures out of n trials inclusive is approximately equal to

$$P(Z \geq (k_1-0.5-np)/\sqrt{np(1-p)}) - P(Z \geq (k_2+0.5-np)/\sqrt{np(1-p)})$$

We have listed the approximations in the form

$$P(Z \geq a)$$

so that the use of Appendix B, Table 2 is direct.

As an example, suppose that $n = 40$ and $p = 0.3$. The probability of between 10 and 20 failures inclusive is

$$P(z \geq (10-0.5-(40)(0.3)/\sqrt{(40)(0.3)(0.7)})$$

$$- P(Z \geq (20+0.5-(40)(0.3)/\sqrt{(40)(0.3)(0.7)}) .$$

Simplifying we obtain

$$P(z \geq - 0.86) - P(Z \geq 2.93).$$

Now from Appendix B, Table 2, we find that $P(Z \geq - 0.86) = 0.8051$ and $P(Z \geq 2.93) = 0.0017$. Consequently, the probability that between 10 and 20 failures inclusive occur is approximately 0.8034.

The value using a set of binomial tables is 0.8017.

A-2. POISSON APPROXIMATION TO BINOMIAL

When p, the probability of failure on a given trial, is extreme ($p \leq 0.2$ or $p \geq 0.8$) and n, the number of trails, is large ($n \geq 30$), the Poisson distribution provides reasonable approximations to binomial probabilities. We make the identification $m = np$ and use Poisson tables to determine the probabilities of events in a binomial experiment.

As an example, suppose that $n = 40$ and $p = 0.05$, so that $m = 40(0.05) = 2$. The probability of between 5 and 10 failures is the difference between the probability of 10 or fewer failures (1 .000) and the probability of 4 or fewer failures (0.947) . (See Appendix B, Table 3.) The difference is 0.053. Using a set of binomial tables we obtain 0.0480.

A-3 . NORMAL APPROXIMATION TO POISSON

When the product AT is greater than 5, the normal distribution provides reasonable approximations to Poisson probabilities. The approximation is detailed below. Note that Z is the notation for the standard normal variable. (See Appendix B, Table 2)

The probability of k or fewer failures during time T is approximately

$$1 - P(Z \geq (k+0.5-\lambda T)/\sqrt{\lambda T}) .$$

The probability of at least k failures during time T is approximately

$$P(Z \geq (k-0.5-\lambda T)/\sqrt{\lambda T}) .$$

The probability of between k_1 and k_2 failures inclusive during time T is approximately

$$P(Z \geq (k_1-0.5-\lambda T)/\sqrt{\lambda T}) - P(Z \geq (k_2+0.5-\lambda T)/\sqrt{\lambda T}) .$$

We have listed the approximations in the form

$$P(Z \geq a)$$

so that the use of Appendix B, Table 2 is direct.

As an example, suppose that the failure rate A is 0.01 and the test time T is 1000 hours. The probability of between 8 and 15 failures inclusive is

$$P(Z \geq (8-0.5-(0.01)(1000)/\sqrt{(0.01)(1000)})$$

$$- P(2 \geq (15+0.5-(0.01)(1000)/\sqrt{(0.01)(1000)}) .$$

The above expression reduces to

$$P(Z \geq -0.79) - P(2 \geq 1.74).$$

Now $P(Z > -0.79) = 0.7852$ and $P(Z \geq 1.74) = 0.049$, so the probability that between 8–and 15 failures inclusive occur is approximately 0.7443.

Using the Poisson tables (Appendix B, Table 3), we obtain the probability more precisely as 0.731.